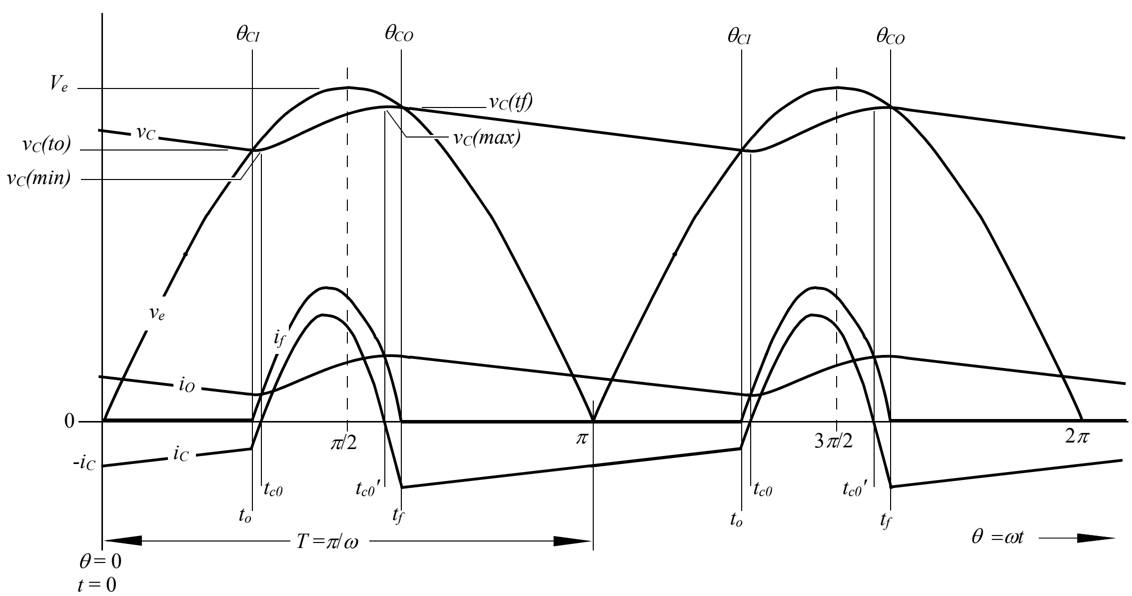
Rectifier Design and Analysis

This document describes the operation and design of AC line operated rectifier circuits. Both theory of operation and detailed design procedures are included, neither of which require complex differential equations for analysis of the transcendental relationships.

While this is not a mathematically "pure" analysis, it is based on the principles of transcendental relationships using graphical integration, initial conditions and solution of discontinuous functions; providing useable results for the actual construction of working systems as well as an intuitive understanding of the voltages and currents involved. The complex mathematical analysis normally required to predict circuit behavior has left many, otherwise technically savoy, individuals relying on handbook data without understanding the actual interactions involved.

This document is written for individuals with a technical understanding of basic electronics.



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Introduction to Rectifier Design

The objective of this document is to provide an intuitive approach to rectifier design that will enable the reader to grasp the complex relationships of voltage and currents in these seemingly simple circuits. Taking this intuitive understanding to the next step, a practical approach is presented to calculate (within a reasonable degree of accuracy) the peak and *rms* voltage and currents necessary to specify transformers, diodes and capacitors for a given application.

Traditionally, Electrical Engineering texts present basic rectifier analysis using transcendental relationships solved with differential equations. [1,2] But, the inclusion of real world conditions can make this analysis worthy of a Doctorial paper. Presented here is an approach that accurately designs rectifier systems producing both useable results and an understanding of the voltages and currents involved while being simple enough to be understood by a person with a basic technical background in electronics. However, this analysis is based on the same transcendental relationships, typical of time variant systems, but the equations presented or either developed from basic electrical laws that the reader can follow or referenced to published technical literature.

Since the reader is assumed to be familiar with basic electronic theory, this document will limit further explanations to rather specific topologies. Considering today's state of the art, the primary requirements covered in this document are for full-wave center tapped (FWCT) and full-wave bridge (FWBR) topologies both feeding into shunt-capacitor filters. This material is **not** to be used to design poly-phase kilowatt power supplies, but rather, be used by those technically competent, to design low to medium power equipment for personal or non-production use. ***Any intent to sell product or design information is subject to local, state and federal laws and in most cases must meet other regulatory requirements.***

Figure 1 is a FWCT rectifier, Figure 2 is a FWCT rectifier with dual outputs and Figure 3 is a FWBR rectifier. Using conventional current flow, i_f is the total forward current through the rectifier diodes [3], i_O is the output current and i_C is the capacitor current. [4] In this document, lower case v and i represent time

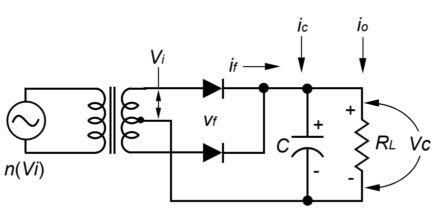


Figure 1. Full-wave center tap rectifier and shunt-capacitor filter schematic.

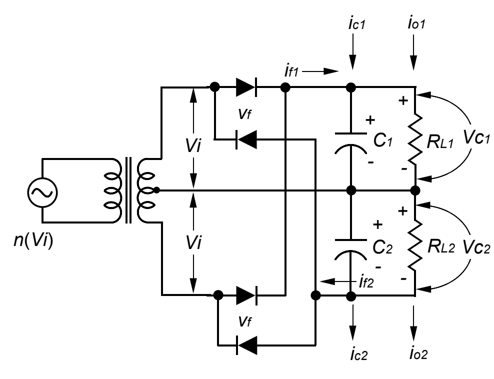


Figure 2. Full-wave center tap rectifier and shunt-capacitor filter with dual outputs.

1 *Basic Electronics for Engineers and Scientists* by Lueg and Reinhard 1972 International Textbook Company
2 *Radio Engineering* by Frederick Terman, Sc.D., 2nd Edition, McGraw-Hill Book Company, Inc. 1937
3 For FWBR this is the current through each diode, but for FWCT each diodes's current is 0.707 times this value.
4 Charge displacement current - see Quantitative Analysis for further discussion.

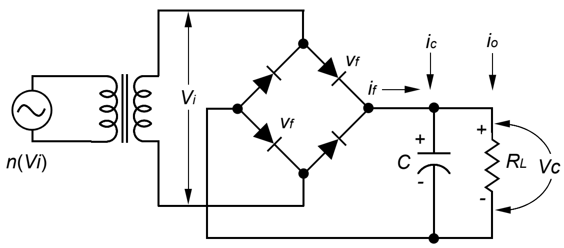


Figure 3. Full-wave bridge rectifier and shunt-capacitor filter schematic.

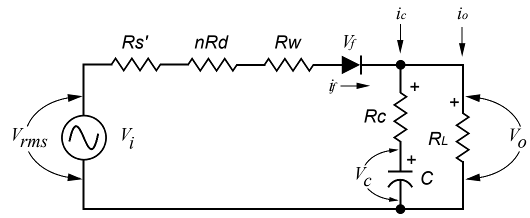


Figure 4. Equivalent circuit for either FWBR or FWCT with appropriate values.

variant values and upper case I and V represent peak or *rms* values, noted as used. V_i which also is V_{rms} is the transformer's secondary *rms* voltage measured from one end of the secondary to the center tap for FWCT or across the entire secondary for FWBR.

Figure 4 is an equivalent circuit applicable to both FWCT and FWBR rectifiers that will be used as the basis of the calculations to follow. The transformer will be studied further, but for now it is assumed to have a turns ratio of $n = n_p/n_s$ with n_p and n_s equal to the number of turns in the primary and secondary windings. The transformer's leakage inductance is L_{LK} and for the majority of this document is assumed to be negligible. [1] The resistance of the transformer's windings is R_s', where R_{ss} is the DC resistance of the secondary winding and R_{sp} is the DC resistance of the primary winding which couples into the secondary at the ratio of n^2 so

$$R_s' = R_{ss} + R_{sp}/n^2.$$

The FWCT rectifier with dual outputs, see Figure 5, is an exception to the common analysis. Note each half of the center tap secondary has its' own unique R_{ss}, assuming L_{LK} is negligible, but the contribution of the primary, R_{sp}, is common to both outputs. If the two loads are always complementary this has no significance; however, if the outputs can be widely different then it is important to note that the output drawing the larger current will reduce the output drawing the lesser current by the voltage drop across R_{sp}/n^2, that is to say the two outputs are mutually coupled by the primary losses.

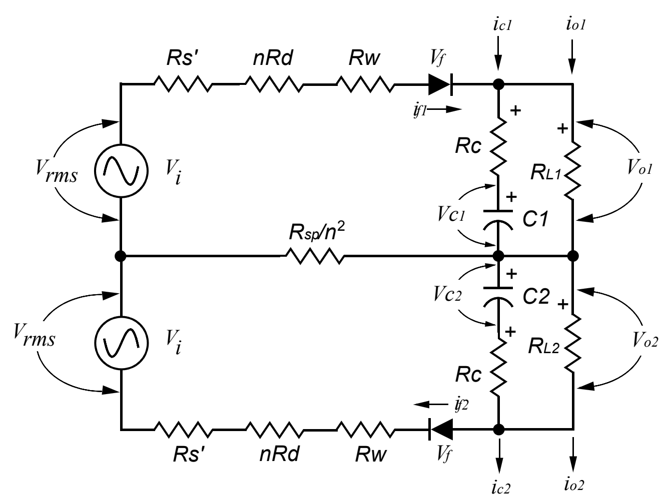


Figure 5. Full-wave center tap rectifier and shunt-capacitor filter with dual outputs including common primary DC resistance.

1 See Appendix F, Components, for additional comments.

Calculations for this topology are carried out for each output just as for a FWCT including R_{sp}/n^2 in each calculation, as normal. When each output's voltage and current values are known, then the effect of each output's current on the other can be calculated by subtracting $(R_{sp}/n^2)(i_{o1})$ from output 2 voltage and visa versa.

The transformer's intrinsic resistance prevents actual measurement of V_{rms} under load, so when monitoring the secondary with an oscilloscope it will appear the output is distorted. In reality this is an artifact of the voltage drop caused by R_s as the pulsating i_f is delivered to the load and filter capacitor. This pulsating secondary current is the major difference in transformer service ratings between resistive load and rectifier service. [1]

Another contributor of series resistance is the forward dynamic resistance (R_d) of the rectifier diodes. The diodes forward drop is subtracted from the peak value of V_{rms} in the calculations, but this value is static at the current level of I_O. Rectifier diode dynamic resistance is the change in voltage as a function of a change in forward current (resistance). With modern silicon and Schottky diodes this value is rather small, but none the less, a contributor and is added to R_s' so that

$$R_s = R_s' + nR_d. + R_w$$

Most manufacturer's data sheets will provide a typical "Forward Characteristics" graph of forward voltage vs current and the slope of the curve is dynamic resistance, that is

$$R_d = \Delta v_f / \Delta i_f .$$

The final contributor to R_s is the wiring resistance, R_w, which is comprised of the total resistance of the conductors interconnecting the transformer, rectifiers and filter capacitor. For low current, high voltage power supplies this contributor may be small, but for medium or high current, low voltage power supplies this is a significant value.

The rectifier diode is represented by v_f, and when on has the resistance and forward drop as stated and when off has no reverse current flow, a reasonable assumption with modern diodes.[2] C is assumed to have a significant *ESR* of R_c, but insignificant *ESL*, leakage current and dielectric absorption. For the calculations to follow this is reasonable, using modern electrolytic capacitors operating at 50 or 60 Hz. *ESR* will be covered further in discussing *rms* ripple current, dielectric heating, and total ripple voltage.

R_L is assumed to be a fixed, pure resistance equal to the maximum steady state load. If an electronic regulator is the load for the rectifier system, then the load is a constant current such that $i_O(t) = I_O$ that is to say the output current is not time variant, but simply equal to the DC current, at least once steady-state conditions are reached. However, in this document the rectifier load is as-

1 See Appendix F, Components, for additional comments.

2 Note that in the following calculations and Figures the diode static forward drop is subtracted from the actual input voltage which then is identified as v_i.

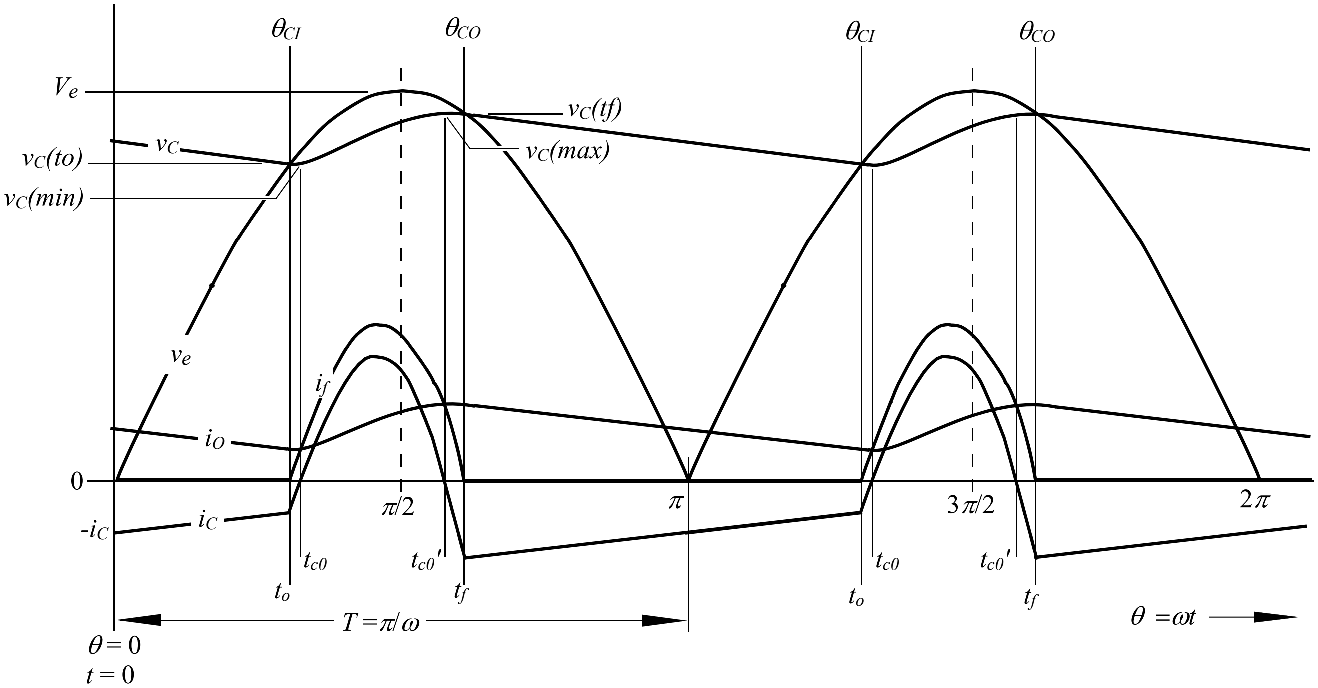


Figure 6. Equivalent Input Voltage, Capacitor Voltage, Forward Diode Current, Capacitor current and Output current versus ωt for a full-wave rectifier with shunt-capacitor filter.

sumed resistive which has minimum effect on the results, except for output ripple current with a constant current load which, as just observed, is near zero because the regulator draws constant current over the range of the ripple voltage.

Qualitative Analysis of Voltages and Currents

In keeping with standard engineering practice, angular relationships with angles expressed in radians are presented in this document, in part to provide consistency with the referenced published works and in part to simplify the analysis. One other point of convention, in Figure 6 voltage and current are portrayed on a scale with a 0 starting point. Actually this is just a reference point in the angular rotation of v_i as it passes through an angle of 0 radians. The actual time from when voltage was first applied is unspecified but it is assumed steady-state conditions are already established. More on this in the Quantitative Analysis where initial conditions are defined and the transit condition from the instant of power application to steady-state is explored.

Figure 6 provides critical insight into understanding a FW rectifier's voltage-current relationships. This busy graph, if studied carefully, will serve one well. The input voltage (v_i) is a rectified voltage of the form

$$v_i = V_m\,|\sin(\omega t + \omega t_o)| - n v_f,\ \text{where } \omega = 2\pi f \text{ and } \pi/\omega = T,$$

which is the **half-cycle** period of the incoming AC line voltage. [1,2] Figure 6 shows this voltage starting at a phase angle of 0 and continuing through to 2π radians, or one complete cycle of the incoming AC line voltage.

For the circuit conditions portrayed in Figure 6, v_e is the driving voltage and is related to v_i by,

$$v_e = \frac{R_L(v_i)}{R_s + R_L}$$

as well, the capacitor already has a charge at $t = 0$, with a resultant voltage (v_C) from which the first basic relationship is seen. Namely, the rectifier will not become forward biased until v_e is equal to, or greater than, v_C . This point in time is designated t_o signifying rectifier turn-on and the initiation of rectifier current flow (i_f) which will increase over time as v_e continues to increase faster than v_C. Some texts also identify this point as θ_{CI}, the cut-in angle of the input voltage that initiates current flow i_f. [3]

At t_o, v_C is near, but not at its' minimum value. v_C will reach its' minimum when i_f becomes equal to i_O, or the instant $i_C = 0$. i_C is negative while supplying the output current, which it does until i_f starts to flow at t_o, but during the transition between t_o and t_{C0} (the point in time at which $i_C = 0$) the capacitor supplies a decreasing amount of the output current until i_f supplies the entire output current. From this point i_f also begins to charge the capacitor, i_C becomes positive and v_C begins to increase. Thus, the minimum ripple voltage occurs not at rectifier turn-on, but a short time latter at t_{C0}.

As i_f begins to flow, it takes on a complex pulse form, that is dependent upon the ratio of R_S/R_L, and the value of C. [4] This current pulse will peak before the incoming voltage does and will last until some time <u>after</u> the incoming voltage peaks, a time designated as (t_f), signifying turn-off of the rectifier - also a point in angular rotation noted as (θ_{CO}) or the cut-off angle.

As might rightfully be surmised, t_f provides another critical point of insight. First, this is the point at which rectifier current stops and the capacitor resumes providing the entire output current. Similar to the condition of v_C at t_o, the peak output voltage precedes this point by a slight amount, because i_f decreases as v_e drops toward v_C . The resultant current flow through the rectifier and source resistance is no longer supplying the entire output current, resulting in some amount of current again being drawn from the capacitor, in turn causing its' voltage to drop. Thus, the maximum ripple voltage occurs not at rectifier turn-off, but rather a short time prior when $i_C = 0$ at time t_{C0}' .

Second, starting at t_f the capacitor will discharge at a rate that can be calculated with the basic exponential decay formula - assuming the load is resistive. If the load is a constant current the capacitor voltage will decay linearly per the basic differential equation relating charge and current in a capacitor. [4] The

1 This is the general expression for an ac voltage with t_o representing time = 0 and should not be confused with the convention in this document of t_o representing the point at which the rectifier diode turns on. Also note, T is $1/2f = \pi/\omega$ and **<u>not</u>** the customary $1/f$.

2 Note that in the following calculations and Figures the diode static forward drop is subtracted from the actual input voltage which then is identified as v_i.

3 *<u>Basic Electronics for Engineers and Scientists</u>* by Lueg and Reinhard 1972 International Textbook Company

4 See Appendix B, Relationship of voltage and currents to R_S, R_L and C.

discharge will continue through time until v_e once again overcomes v_c at which point the process repeats, at the next t_o.

And lastly, note v_c does not perfectly track v_e during the time the rectifier is on. This is because of the series resistance described earlier and is more pronounced the larger R_s, or if L_{LK} is significant. Actually, v_c can **theoretically** track v_e if $R_s = 0$ (a source of infinite power), but the equation presented in this document will not calculate i_f for this condition as it causes a divide by 0 operation. Not to worry though, this is an impossible real world condition and if curiosity demands a solution, differential equations are presented in engineering texts, for instructive purposes, solving such a theoretical condition. [1]

Handbook Design Factors

FWBR or FWCT rectifier circuit design can be simplified by starting with some rough estimates of performance. These can then be used to make more specific calculations that will determine actual specifications for the key components.

Table 1. First, consider the trade-offs between FWBR and FWCT circuits operating into a shunt-capacitor filter. [2]

Design Consideration	FWBR	FWCT
rms secondary current	$1.65I_o$	$1.15I_o$
peak secondary current	identical, but FWCT *rms* current is less than FWBR by the √2	
Secondary voltage	V_o(0.7 to 0.95)*	V_o(1.4 to 1.9)*
Rectifier diodes PIV	1.41	2.82
Transformer utilization**	Poor	Poorer
Voltage regulation	Poorer	poor

* Depending upon load current.
** High peak to average ratio of currents in shunt-capacitor filter systems cause poor utilization of the transformer and poor power factor - see text for additional information.

The selection of topology may not be possible until after detailed calculations are made. However, some guidelines are; FWCT requires a transformer with almost twice the secondary voltage, the secondary *rms* current is only less by the √2 and peak secondary current is the same as for FWBR, with primary current very similar for either. Even though the FWCT also requires the complication of a center tap connection, it may still be preferable in high current rectifiers where the additional diode's power loss and voltage drop are more significant than the associated transformer complications.

1 *Basic Electronics for Engineers and Scientists* by Lueg and Reinhard 1972 International Textbook Company
2 *Reference Data for Radio Engineers* Sixth Edition 1982 Howard W. Sams & Co., Inc.

First Pass Approximations

In order to work through the detail calculations, it is necessary to estimate appropriate values for some of the unknowns, a task not as difficult as it might first seem. Start with the desired output voltage, I will not belabor the point on how to select this value, but some guidelines are;

1) if the output feeds an electronic regulator, the minimum ripple voltage must remain greater than the drop-out voltage at minimum line voltage and full load, but the greater the *rms* output voltage the greater the power dissipation and lower the efficiency;
2) if the output will be used directly or minimum ripple voltage is desired, it is primarily dependent upon load resistance and the capacitor - to the extent the source resistance can charge the capacitor;
3) the voltage regulation is primarily dependent upon the load resistance and the source resistance and only slightly on the capacitor;
4) as well, the peak current is also primarily dependent upon the load and source resistance so improving regulation will increase peak current. [1, 2]

$$C \cong \frac{1}{\ln\left|\frac{1}{1 - v_r/v_c}\right| \frac{\omega R_L}{0.6\pi}} \qquad (1)$$

and,
$$R_s < \frac{0.4\pi}{\omega C} \qquad (2)$$

In a similar fashion, select the full-load output current. If the load is an electronic regulator this will be a constant current. If the output is used

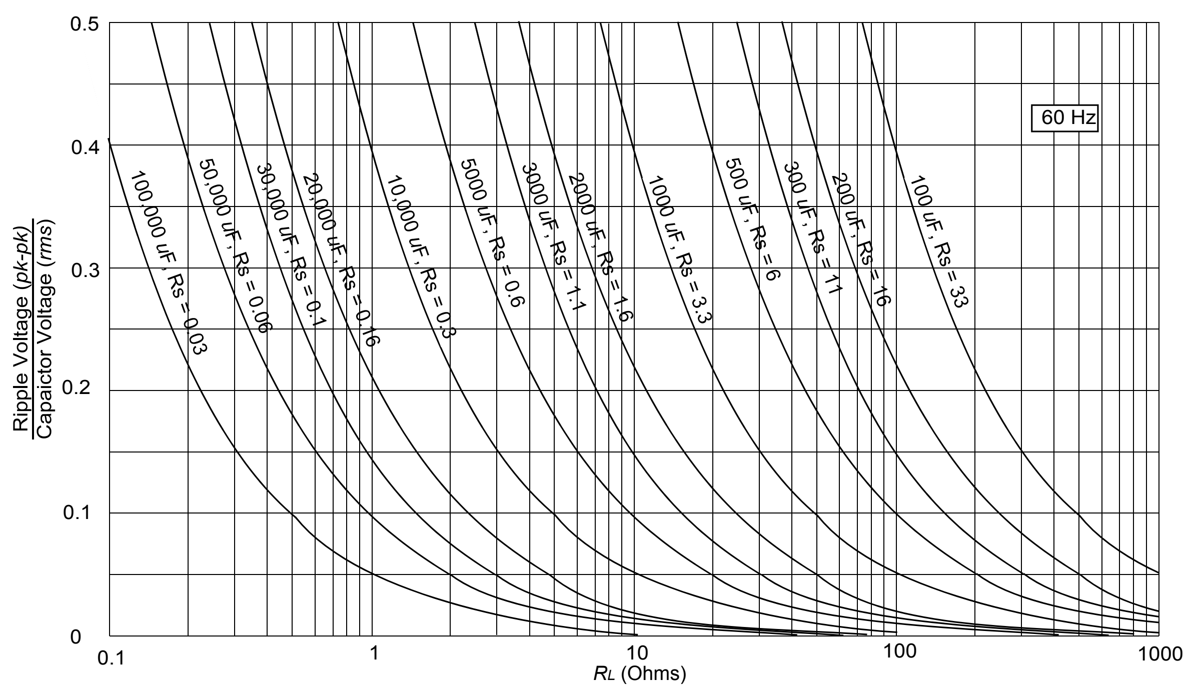


Figure 7. Graph for estimating the capacitor value and maximum series resistance, given a desired ripple voltage (peak-to-peak) and load resistance. Graph is for power line frequency of 60 Hz.

1 *Radio Engineering* by Frederick Terman, Sc.D., 2nd Edition, McGraw-Hill Book Company, Inc. 1937
2 See Appendix B for further discussion.

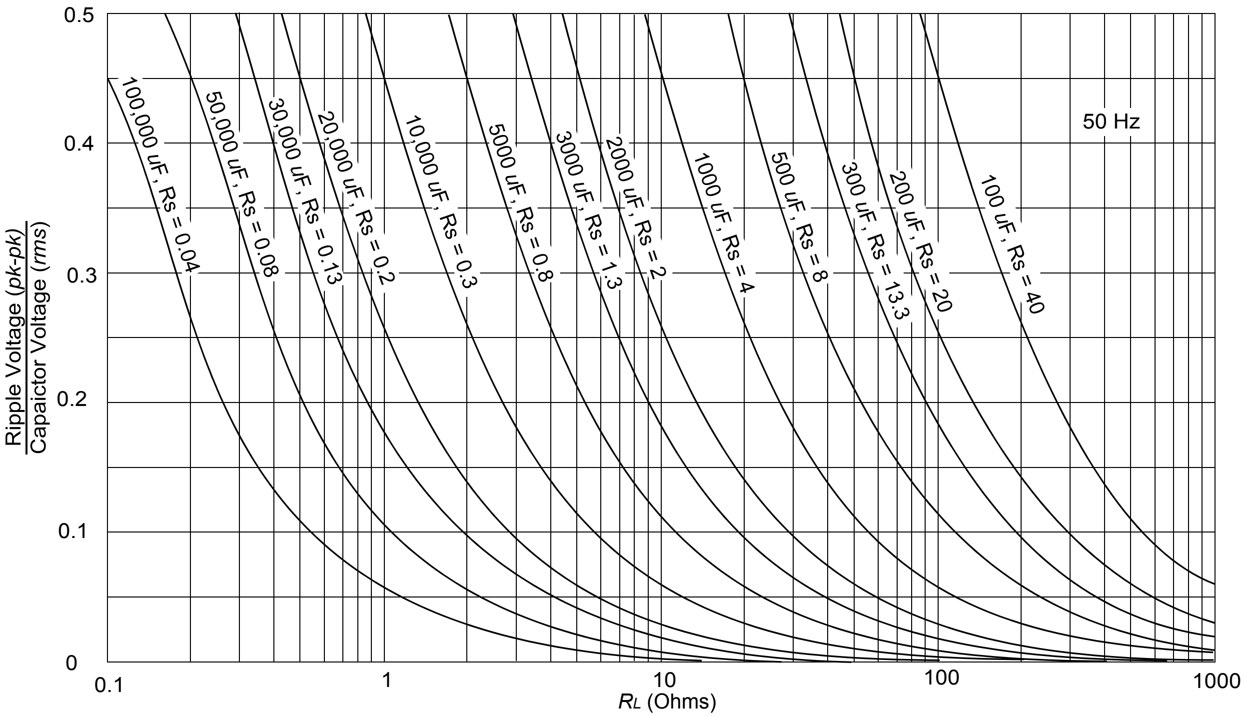


Figure 8. Graph for estimating the capacitor value and maximum series resistance, given a desired ripple voltage (peak-to-peak) and load resistance. Graph is for power line frequency of 50 Hz.

directly, without electronic regulation, assume the load to be a resistance equal to the desired *rms* output voltage divided by the full load output current.

Selection of the shunt-capacitor will be finalized after the *rms* ripple current is calculated, as this often is the controlling factor in it's value. As a first approximation select a capacitor that has a working voltage somewhat higher than V_m with the AC line voltage at its' maximum value. Refer to specific manufacturers data for guidance.

The starting point capacitance can be estimated by Equation (1) or Figure 7 (60 Hz) or Figure 8 (50 Hz). [1] This equation is based on selecting a desired ripple voltage (peak-to-peak), output load resistance (at full load), *rms* output voltage at full load, and line frequency. The estimation is based on the assumption that the discharge time is 0.6 of a half-cycle period - which is sufficient to get started, remember the final value will be selected after detail calculations. This also provides a starting point for selection of the transformer by establishing a **maximum** value for R_s using Eq. (2) or again reading R_s from the capacitance curves of Figure 7 or 8. If these values seem unreasonable, then adjust them accordingly and then run through the detail calculations to rationalize.

[1] See Appendix A for derivation of these equations.

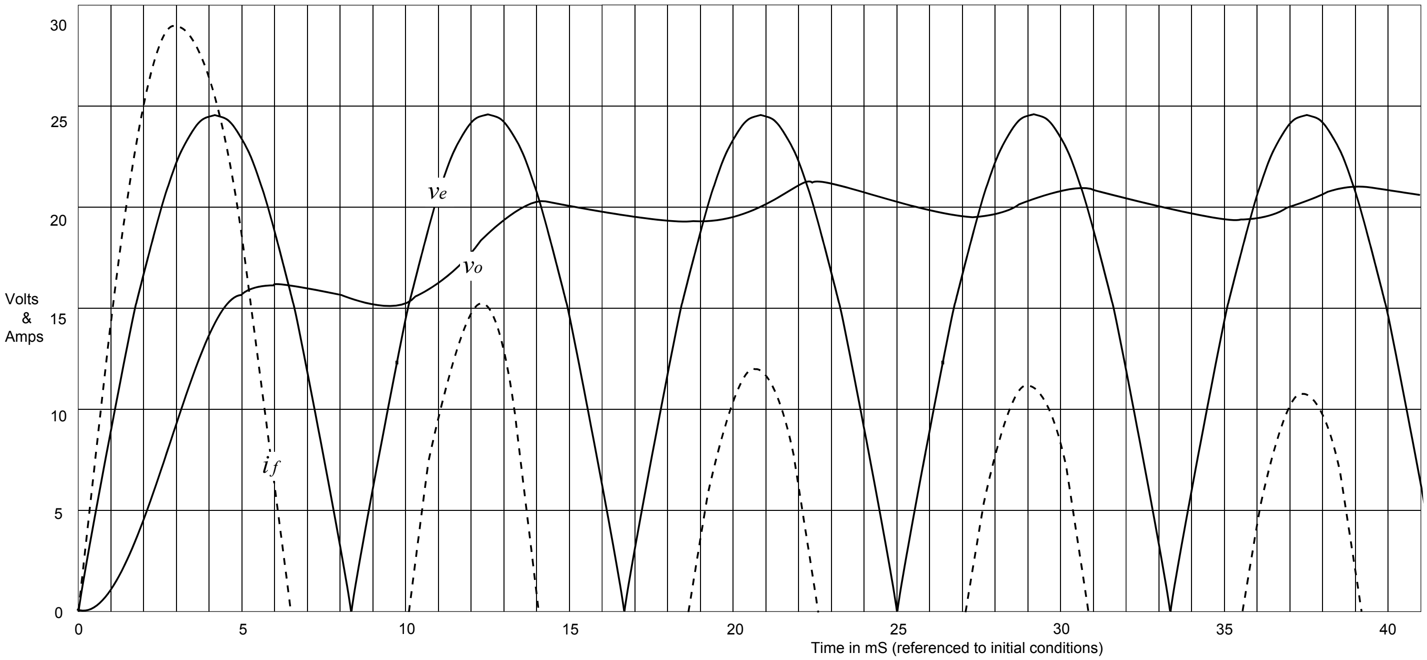


Figure 9. Relationship of voltages and currents over five half-cycles starting with initial conditions of $v_e = v_C = t = 0$, for example in Fig. 10.

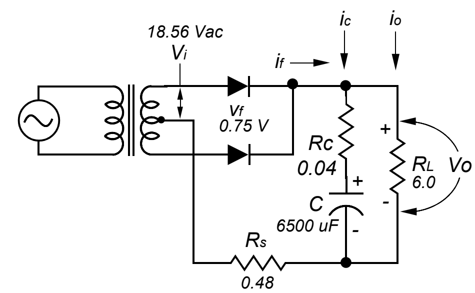


Figure 10. Example rectifier with $R_s = 0.51\ \Omega$.

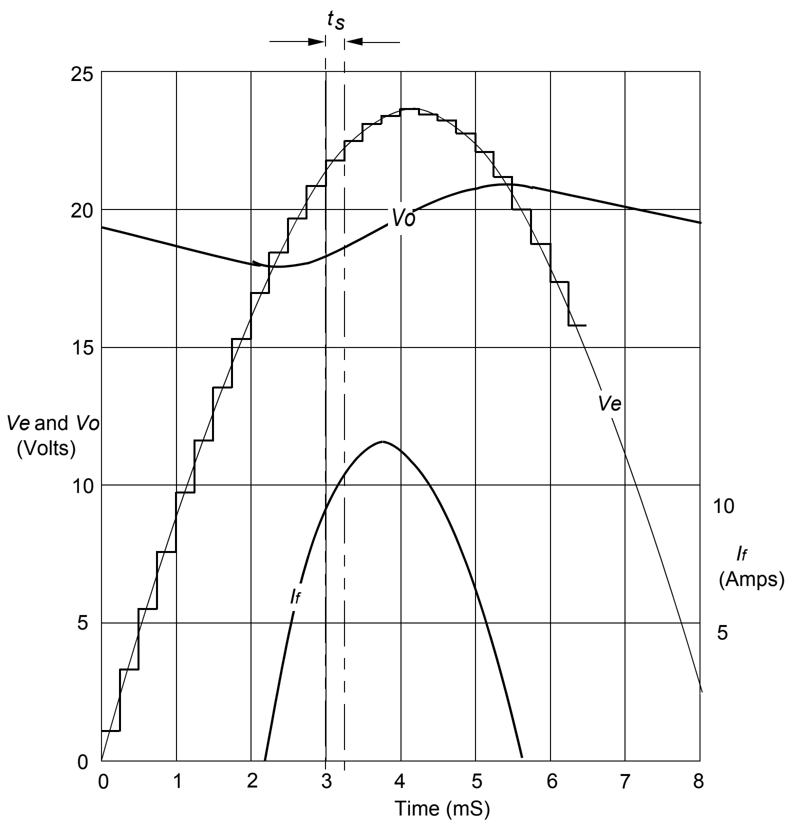
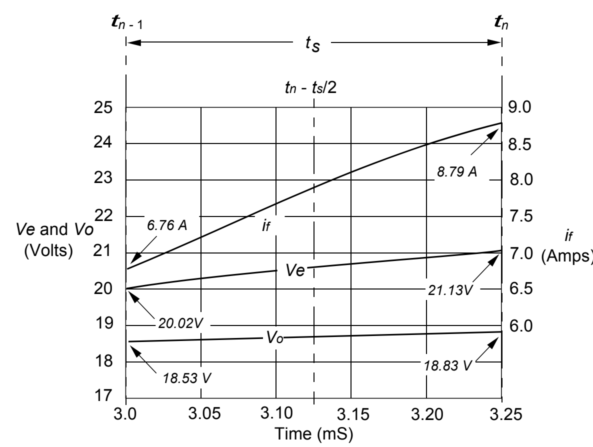


Figure 11. Equivalent circuit input voltage shown quantized into discrete 0.25 *mS* voltage steps. Numerical data for example in Figure 10, after steady-state conditions are established.



Firgure 12. Detail of the step in Figure 11 from 3.0 to 3.25 mS. Expanded view of voltages and currents over the duration of the step.

Quantitative Analysis

Initial Conditions and Surge Currents

In order to proceed with detail calculations, initial conditions must be defined. Even though these can be set at random, some advantage is had by using the initial conditions shown in Figure 9. The AC input voltage alternately cycles from a negative peak to a positive peak and again to a negative peak, crossing through 0 volts each direction. After full-wave rectification it has the unipolar form shown in Figure 9. If we pick a point on this rectified input voltage where it is going positive from 0 volts and define it as t = 0 we have a logical starting point for initial conditions, especially if we let $v_C = 0$ at this point. By Ohms law, if $v_i = v_C = 0$, then all currents are also 0.

So the instant power is applied the natural response of the rectifier circuit is for current to flow like it would in any series *RC* circuit. Figure 9 shows this as i_f leads v_C and increases to a value well above steady-state conditions. However the total circuit response is also influenced by the forced component of the rectified driving voltage. That is to say, if v_i was a DC voltage the capacitor would continue to charge, at a rate determined by the R_sC time constant, until fully charged. But since the forcing voltage is rectified AC, there may be insufficient charge accumulated on *C* during this first half-cycle to establish steady-state conditions. This is seen as v_C increases but does not reach steady-state during the first half-cycle.

The second half-cycle again charges *C*, this time from a starting voltage much above 0 and so by cycle's end v_C is much nearer its' steady-state value than it was at the end of the first half-cycle. This transit response is a function of the circuit values and independent of the forcing voltage, depending on the actual values it may last several cycles, such as the example in Figure 9 takes five half-cycles to reach equilibrium (steady-state).

By starting calculations with these initial conditions we capture yet another important value - the repetitive surge forward current that may be expected for severe over-loads. For rectifiers that feed an electronic regulator, this is less important than for a rectifier that feeds a non-current limited load. Normally an electronic regulator will manage output overloads, but if the rectifier does not have such protection it will be possible for severe overloads to pull v_C to 0 Volts, or nearly so. This will cause a repetitive surge forward current pulse each half-cycle that is similar to initial turn-on.

This is not to be confused with non-repetitive forward surge current (I_{FSM}) that semiconductor manufacturers normally specify for a single cycle of either 50 or 60 Hz. If a forward current of this magnitude becomes repetitive the diode junction will be destroyed, whereas the repetitive surge forward current's *rms* value must not exceed the diodes *rms* forward current rating (I_{Frms}), also denoted as (I_O) by some manufacturers.

Worst case I_{FSM} is limited only by the equivalent series reactance (resistance and leakage inductance) of the transformer, the rectifier resistance and the capacitor's ESR. Refer to Figure 9 to visualize a case where the initial conditions have the input voltage at its' maximum value rather than at 0, for this example a little over 25 Volts. For an ESR of 0.01Ω and a R_S of 0.51Ω the

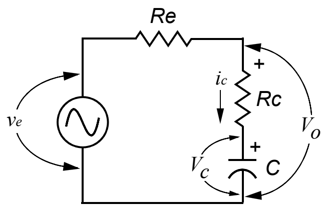


Figure 13. Equivalent circuit of Figure 4 with R_S (R_S' + nR_d + R_W) and R_L combined into R_e and rectifier on, that is, during the angle of conduction θ_{CI} to θ_{CO}. Also note v_e replaces v_i according to Equation (5).

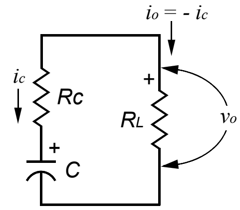


Figure 14. Equivalent circuit of Figure 4 with R_S removed, that is the rectifier off during the angle of θ_{CO} to θ_{CI} . Note during the rectifier off time, the circuit is a simple *RC* circuit as the discharge of *C* supplies the entire output current.

surge current is almost 50 Amps. This current surge will decay quickly lasting only a fraction of the AC cycle, i.e. limited by the time constant of R_SC, but even so, it is critical that it not exceed I_{FSM}. It is prudent to include an appropriately sized NTC thermistor, known as "inrush current limiter" in the primary side of the transformer to reduce this inrush current to a safe value.

In order to determine the current and voltages in Figure 9 it is necessary to make repeated calculations of forward current, i_f , starting at the initial condition and for each step thereafter until the first half-cycle t_f is reached, using Equations (3) through (11).[1]

First, the equivalent circuit of Figure 4 is reduced by Thevenin's theorem to Figure 13, using Equations (3) through (6).

(3) $$R_e = \frac{R_s R_L}{R_s + R_L}$$ (4) $$R_s = R_s' + nR_d + R_w$$ (5) $$v_e = \frac{R_L(v_i)}{R_s + R_L}$$

(6) $$v_i = \sqrt{2}V_i\,(rms)|\sin(t - t_s/2)| - nv_f$$

Figure 13 represents the rectifier circuit during the time forward current flows through the diodes, that is during the angle of conduction from θ_{CI} to θ_{CO}. Equation (7) is an approximation of the differential equation for i_c

(7) $$i_c(t)\Big|_{\theta_{CI}}^{\theta_{CO}} = \frac{v_e(t_n) - v_c(t_n - 1)}{R_e + R_c}$$

where v_e is the driving voltage, calculated at the midpoint of the step from (t_n -1) to t_n, and v_c(t_n -1) is the voltage on the capacitor at the beginning of step t_n. The forward current is time dependent, that is its' value depends on v_c (which is also time dependent) at each instant of time. For this reason the calculations are made using quantized steps, see Figure 11 and 12, to

1 A similar approach, not developed in detail, is presented in *Circuits, Devices, and Systems* Second printing by Ralph J. smith John Wiley & Sons inc., New York

calculate discrete equivalent values for i_f. Note the restriction that $t_s < R_sC/2$, which is necessary for the equation to track the rate of change of the AC input voltage.

This yields a value at only one point in time and in order to calculate a value at the next step in time (which must be done sequentially) the approximation of the time integral of v_c must be calculated and added to $v_c(t_n\text{-}1)$ to obtain the new $v_c(t_n\text{-}1)$ for the next step of n. This is done using Equation (8) after which the other time dependent variables are calculated using Equations (9) through (11).

(8)
$$v_c(t)\Big|_{\theta_{CI}}^{\theta_{CO}} = v_c(t_n\text{-}1) + \frac{(i_c(t_n))t_s}{C}$$

(9)
$$v_o(t)\Big|_{\theta_{CI}}^{\theta_{CO}} = (i_c(t_n))R_c + v_c(t_n)$$

(10)
$$i_o(t)\Big|_{\theta_{CI}}^{\theta_{CO}} = (v_o(t_n))/R_L$$

(11)
$$i_f(t)\Big|_{\theta_{CI}}^{\theta_{CO}} = i_c(t_n) + i_o(t_n)$$

So, the process is to calculate an instantaneous forward current and integrate the capacitor charge over the time of the step to establish the conditions to calculate the next step, continuing the process until t_f is reached.

After t_f, the capacitor voltage is found using Equation (14), once Equations (12) and (13) are solved.

(12)
$$i_c(t)\Big|_{\theta_{CO}}^{\theta_{CI}} = \text{-}i_o = \text{-}\ \frac{v_c(t_n\text{-}1)}{R_c + R_L}$$

(13)
$$v_o(t)\Big|_{\theta_{CO}}^{\theta_{CI}} = \frac{(v_c(t_n\text{-}1))R_L}{R_c + R_L}$$

(14)
$$v_c(t)\Big|_{\theta_{CO}}^{\theta_{CI}} = v_c(t_n\text{-}1) - \frac{(i_c(t_n))t_s}{C}$$

Provided with this document is an MS Excel® spreadsheet that automates the process and provides summary data, *rms* values and plots for the major parameters. While a programmable calculator can be used, it is very tedious to solve the basic 8 equations for 200 steps! If access to Excel is limited, Java has a free version under OpenOffice.org Calc that is compatible with Excel. Once the theory and equations are understood little is lost using the spreadsheet which makes it very easy to optimize a design by running numerous iterations.

Calculations should continue until convergence is obtained, or simply until v_c at t_o and t_f are within a few percent of one another respectively, from the last and next to last cycle values. This is the steady state values of voltage and current (for a fixed load and line). If these meet the initial design requirements

$i_f(t)$ = Forward diode current at time t
n = 1 for FWCT or 2 for FWBR
v_f = each diode forward voltage drop at nominal i_o
R_d = dynamic resistance of diodes
R_c = capacitor resistance (*ESR*)
R_s' = xfmr secondary DC resistance + $1/n^2$ (primary resistance)
t_o = the time at which the diodes turn on
t_f = the time at which the diodes turn off

Trig functions in radians - for trig functions in degrees multiply ω by 57.3 (degrees / radian).
$\omega = 2\pi f$ (radians per second)

These equations involve many time-variant values written with *italic parenthesis* not to be confused with the operation of multiplication written with standard parenthesis, for example;
$(v)(t)$ = the value of v multiplied times the value of t, whereas,
$v(t)$ = voltage at time t.
In most instances the meaning is clear due to context, but if doubt exists use this guide.

you are ready to proceed to calculating *rms* values and finalizing component values, otherwise use the data to modify the insufficient parameter and repeat the calculations for the new values. Note the spreadsheet calculates 6 half-cycles using 0.25 mS steps for 60 Hz and 0.30 mS steps for 50 Hz, with the last half-cycle used to calculate *rms* values.

RMS Calculations

To specify the transformer, diodes and capacitor the *rms* values of voltage and current are needed and can be calculated from the data generated by Equations (3)-(14).

As a note, in theory no current flows "through" the capacitor; but in reality, the charge displacement current that occurs as C charges and discharges causes heating as it encounters the *ESR* (equivalent series resistance of the capacitor). The internal heating caused by this *rms* ripple current is one of the most important considerations in sizing power supply filter capacitors. This internal heat is difficult to dissipate due to the poor thermal conductivity of capacitors. This results in drying out the capacitor's electrolyte, which occurs at a rate proportional to the internal temperature.

Refer to manufacturer's data sheets to determine an appropriate capacitor (or capacitors) for the calculated *rms* capacitor current. Note, *ESR* and *ripple current* capability vary with both frequency and temperature so be observant of these when studying the data sheets. Also the projected operational life of the capacitor is normally predicted based on the percentage the operating ripple current is of the maximum specified ripple current and ambient temperature, so study this for your design as well.

In order to increase the surface area for cooling, either the physical size of the capacitor must increase or the number of parallel capacitors must increase. Some trade-offs are possible here and sometimes several smaller capacitors can be paralleled to produce an acceptable, lower cost filter. Use caution that the capacitors so used are from the same manufacturer and type series to minimize current sharing issues. Note that in Equations (3) through (14) the capacitor's *ESR* is included as R_c because it also has a significant effect on overall circuit behavior in low voltage, medium to high current rectifiers.

As has been seen in the preceding description, the voltages and currents in a rectifier circuit are of a complex form and not readily converted from peak to *rms* with a simple multiplier like 1.41. Therefore to calculate *rms* values, the previous data are graphically integrated to obtain Amp-sec and Volt-sec (that is the area under the curve) and divided by the abscissa (time) over which the integration is specified to obtain a steady-state (DC) value equivalent to the complex form. Equation (15) is the graphical integration of i_f, a set of data for which an exact function is not known, or readily available. A complete explanation of this technique is beyond the scope of this document, so suffice it to say the area under a curve is approximated by this process. [1]

[1] See Appendix A for further discussion.

Equations for calculating *rms* values.[1]

(15)
$$I_f = \frac{(\sum i_f)\cdot(t)}{2n}\sqrt{\frac{nt_s\omega}{\pi}}$$

(16)
$$I_o = \frac{(i_o)\cdot(t)}{T}$$

(17
$$I_c = \frac{(i_c)\cdot(t)}{T}$$

(18)
$$V_c = \frac{(v_c)\cdot(t)}{T}$$

$nt_s = t_n - t_o$, n = number of steps in this context

Figures 15 through 19 will help visualize the process of integration using the "trapezoidal rule" where sufficiently small increments of the abscissa are used to evaluate the corresponding ordinate value, all of which are summed to approximate the area under the total curve. [1]

For continuous functions of time, such as i_c, i_o and v_c (v_o) this equivalent "DC" value will exist over the entire half-cycle (T) and therefore will be the *rms* value of the function. However, for non-continuous functions such as i_f another step is required to obtain *rms* values. Refer to Figures 15 and 16 to visualize a rectangular pulse that represents an equivalent value of the complex form that has a duration of t_n - t_o as a fraction of the total period (T). The *rms* value for such a rectangular pulse is given by Eq. 15. [2]

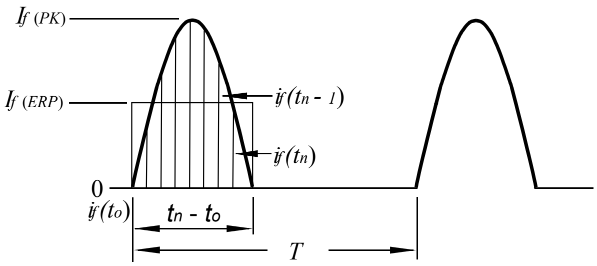


Figure 15. Transformer secondary current for FWBR, showing the conversion of a non-rectangular pulse to a rectangular pulse to determine peak and *rms* value.

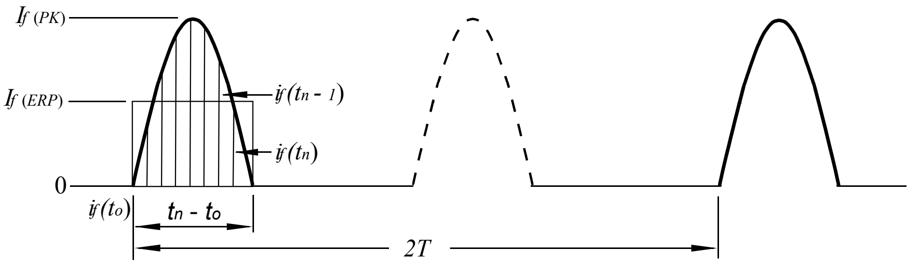


Figure 16. Same as Fig. 14, showing the lower duty cycle of the transformer secondary for a FWCT and the rectifier current for either FWCT or FWBR.

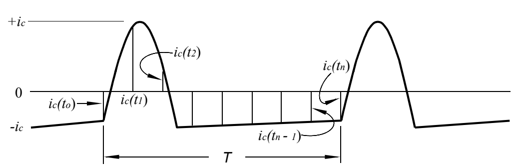


Figure 17. Graphical integration of i_c to obtain equivalent *rms* value.

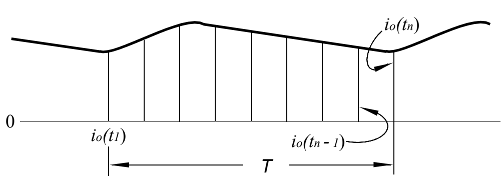


Figure 18. Graphical integration of i_o to obtain equivalent *rms* value.

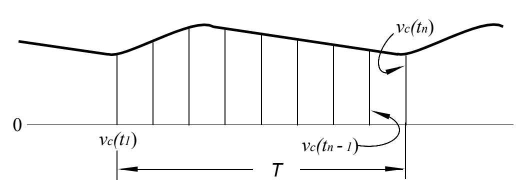


Figure 19. Graphical integration of v_C to obtain equivalent *rms* value.

1 Figures 15 through 19 have only representative time steps.

2 See Appendix A for further explanation.

1 See Appendix A for derivation.

Figure 15 represents the secondary current of a transformer in the FWBR configuration. Note the secondary supplies a current pulse each half-cycle as the bridge diodes connect the correct secondary winding polarity to the load each half-cycle. In other words, the secondary supplies two current pulses each cycle of the AC voltage.

Figure 16 represents the secondary current of a transformer in the FWCT configuration. Note the secondary supplies a current pulse once each cycle as opposed to each half-cycle of a bridge rectifier and therefore has an *rms* current only $1/\sqrt{2}$ times the current of a FWBR connected transformer secondary.

Also, note Figure 16 represents the current pulse through the rectifiers of both a FWCT and a FWBR configuration. For a FWCT this is obvious, but maybe not so in the bridge rectifier. In a bridge rectifier, one pair of diodes are on during each current pulse, but each pair only conducts once each cycle same as a FWCT. So the *rms* **diode** current is identical for equivalent FWCT and FWBR configurations, even though the bridge rectifier has twice the power loss, due to two diodes conducting each cycle.

One last observation, note the peak secondary current is the same for equivalent FWCT and FWBR configurations and the FWCT *rms* current is only less by $1/\sqrt{2}$ due to the duty cycle reduction. So, when considering configurations the FWBR will have twice the rectifier loss, but the FWCT transformer must have twice the total secondary turns, but can **not** have half the wire size (for equivalent R_s).

Component Selection

Selection of components is possible after completing the preceding analysis. Some observations are now presented to assist the designer, but in no way are complete and therefore require considerable understanding and effort on the designer's part.

As previously discussed, the rectifier diodes must be capable of safely handling both the *rms* and surge currents, but several other key characteristics must also be considered. Refer to the manufacturer's data sheet to study the characteristics that a designer must consider. Presented here are considerations for a 50 or 60 Hz rectifier only - for SMPS, or higher frequency rectifiers, numerous other characteristics must also be considered.

Obviously the reverse breakdown voltage rating must be observed for the particular configuration. Several methods are used to characterize this rating and are referred to variously as V_R , V_{RRM} , V_{RWM} and V_{RSM}. These each have specific meanings that a designer needs to understand, best done by studying the specific data sheets of potential devices. Keep in mind that some design margin is needed for high line voltage and no load (even if this is abnormal), as well as for power line induced voltage transients. While beyond the scope of this document the reader is advised to consider techniques to reduce these transients using MOVs, gas-discharge tubes and power line-rated capacitors.

Similar ratings must be observed for the previously discussed forward current - note restrictions also apply to the reverse current. Again study the data sheet carefully to understand how these currents are specified and under what conditions they apply, i.e. duty cycle, temperature, type load, etc.. Be absolutely sure that I_{FSM} or non-repetitive surge current will never be exceeded. Refer to the discussion on page 12 and ensure that external surge current limiting is sufficient to prevent exceeding this rating. Also, be observant that diodes normally have a transient thermal response that must be considered for short-term overloads.

Considering the thermal aspects, be sure to understand and allow for maximum junction temperature by de-rating appropriately for the worst-case ambient temperature, air flow and density (altitude), humidity and thermal path from junction to ambient. Also, pay particular attention to packages with multiple diodes, e.g. dual TO-247 or four-diode bridge packages, as they have individual diode ratings, but also a total package rating that must be observed.

Filter capacitor selection is covered on page 15, but it is worth noting here that the most common misapplication of aluminum electrolytic capacitors in power supply designs involves the *rms* current rating. A design that exceeds this rating will usually not exhibit any particular distress during the early operational life of the power supply, especially in applications of low duty cycle and intermittent use. However, as previously discussed the internal heating will dry out the electrolyte leading to premature capacitor failure. Any acceptable design must include sufficient de-rating that ensures the capacitor's *rms* current rating is not exceeded, even under worst case conditions.

The transformer design is a topic of its' own and certainly beyond the scope of this document. But again, some observations are offered with the suggestion the reader study this topic at a level sufficient to support the specific design level required.

Some general observations of transformers used in shunt-capacitor rectifier circuits;

1) the transformer utilization is poor for FWBR and even poorer for FWCT - so do not be surprised that a transformer used to produce equivalent power in this application is much larger than if used with a resistive load,
2) according to the laws governing magnetic devices, the transformer ratings are based on *VA* or apparent power and not the actual usable output power - which means more transformer heating in this application compared to a resistive load of equivalent power,
3) transformers are usually designed for either a specific temperature rise or a particular voltage regulation and often require additional characterization for rectifier duty,
4) catalog transformers are almost always specified operating into a resistive load,
5) even transformers specified for rectifier service are often specified operating into inductor input filters because (as

seen in this document) to specify performance into shunt-capacitor filters is rather complex,

6) the power factor of these topologies is almost as poor as the transformer utilization - and indeed has been scrutinized and regulated by international listing agencies and governments.

Some practical transformer characterization techniques are offered in Appendix F.

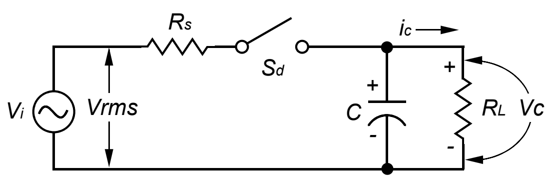
Appendix A

Derivation of Equations

The following derivations of equations used herein that are from basic electrical laws. Understanding these derivations is not required to use the preceding text, but doing so will help the reader comprehend and validate the methods described.

Equation (1). $$C \cong \frac{1}{\ln\left|\frac{1}{1 - v_r/v_c}\right| \frac{\omega R_L}{0.6\pi}}$$

Equation (1) is actually an estimation for reasonable starting component values of a practical rectifier circuit that has a charge time of approximately $0.4T$ and discharge time of approximately $0.6T$. Since these values are only used to start the detail calculations, this estimation is sufficient. Also assume that R_s is sufficiently small compared to R_L that the charging time can be estimated independent of R_L.



C discharging.

General Decaying Exponential Function[1]
$a = Ae^{-t/\tau}$, τ = time constant

v_c decays exponentially with S_d open (rectifiers off) from the value at t_f (the time the switch opens) until t_o (the time S_d closes again), so let $v_r = v_c(t_f) - v_c(t_o)$, that is the peak-to-peak ripple voltage, so

$$v_c(t_o) = v_c(t_f)e^{-t_{off}/R_LC}$$

$$v_r = v_c(t_f) - v_c(t_f)e^{-t_{off}/R_LC}$$

$$v_r = v_c(t_f)(1 - e^{-t_{off}/R_LC})$$

$$\frac{v_r}{v_c(t_f)} = 1 - e^{-t_{off}/R_LC}$$

$$e^{-t_{off}/R_LC} = 1 - \frac{v_r}{v_c(t_f)}$$

solve for C with $t_{off} = 0.6\,T$ and $T = 1/2f$ or in terms of radian frequency, ω

$$\omega = 2\pi f,\ f = \omega/2\pi \qquad T = \frac{1}{2(\omega/2\pi)}$$

$$T = \pi/\omega \qquad \text{and } t_{off} = 0.6\pi/\omega$$

$$C = \frac{t_{off}}{\ln\left|\frac{1}{1 - \frac{v_r}{v_c(t_f)}}\right| R_L}$$

$$C = \frac{0.6\pi/\omega}{\ln\left|\frac{1}{1 - \frac{v_r}{v_c(t_f)}}\right| R_L} \times \frac{\frac{1}{0.6\pi/\omega}}{\frac{1}{0.6\pi/\omega}}$$ simplifies to,

$$C = \frac{1}{\ln\left|\frac{1}{1 - \frac{v_r}{v_c(t_f)}}\right| \frac{\omega R_L}{0.6\pi}}$$

since this is a first-pass approximation, let $v_c(t_f)$ be the desired *rms* output voltage and indicate this equation is an approximation, so

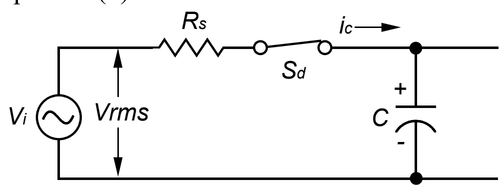
$$\boxed{C \cong \frac{1}{\ln\left|\frac{1}{1 - v_r/v_c}\right| \frac{\omega R_L}{0.6\pi}}}$$

[1] *Circuits, Devices, and Systems*, 2nd Printing 1968, Ralph J. Smith, John Wiley & Sons, Inc.

Rectifier Design and Analysis

Appendix A
Derivation of Equations

Equation (2).



C charging.

$$R_s < \frac{0.4\pi}{\omega C}$$

t_{off} selected for Equation (1), determines t_{on} as,

$t_{on} = T - t_{off}$, with $t_{on} > R_sC$ and rearranged, $R_sC < \pi/\omega - 0.6\pi/\omega$

$$\boxed{R_s < 0.4\pi/\omega C}$$

Equation (17).

$$(i_c)\cdot(t) = \int_{t_0}^{t_n} f(i_c)\,dt \;\cong\; \frac{t_s}{2}\overbrace{\left(|i_c(t_o)| + 2|i_c(t_1)| + 2|i_c(t_2)| + \cdots 2|i_c(t_{n-1})| + |i_c(t_n)|\right)}^{\sum i_f(t)}$$ [1]

Refer to Figures 15 and 16 in order to visualize how the above approximation can determine the area under the current pulse. i_f (ERP) represents *equivalent rectangular pulse* Amp-secs of this pulse and is in a form that can be converted into an *rms* value.

$(i_c)\cdot(t) = \frac{t_s}{2}\sum i_f(t)$ (Amp· secs) $nt_s = t_n - t_o$, where n = number of steps which are one less than the number of values

$I_f = \frac{t_s}{2}\sum i_f(t) \div (t_n - t_o)$ (Amp· secs)/secs

$I_f = \frac{t_s}{2}\sum i_f(t) \div (nt_s)$ Equivalent rectangular pulse Amps

$$I_f = \frac{t_s}{2}\,\frac{1}{(nt_s)}\sum i_f(t)$$

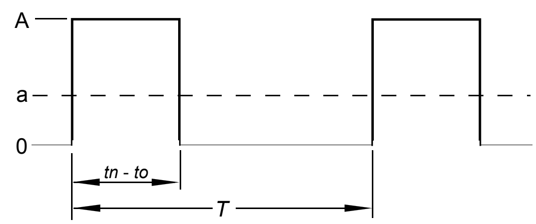
simplify and rearrange

$$I_f = \frac{\sum i_f(t)}{2n}$$

$$\boxed{I_f\,(rms) = \frac{\sum i_f(t)}{2n}\sqrt{\frac{nt_s\omega}{\pi}}}$$

converts rectangular pulse to *rms*

$$a_{rms} = A\sqrt{\frac{t_n - t_o}{T}}$$



Relationship between a rectangular pulse peak and *rms* value.[2]

1 *The Calculus*, Reprinted 1971, C. O. Oakley Barnes & Noble, Inc. Trapezoidal Rule of approximate integration.

2 *Reference Data for Radio Engineers* Sixth Edition 1982 Howard W. Sams & Co., Inc.

Rectifier Design and Analysis

Appendix B

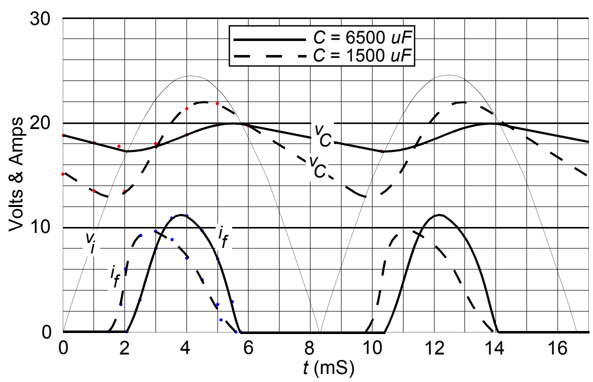
Related Equations and Discussion

$v_c(t) = v_c(t_f)e^{(t_f\ -\ t)/R_LC}$ Exponentially decaying capacitor voltage with fixed resistance load between (t_f - t). Same basic law as Equation (1).

$v_c(t) = v_c(t_f) + \dfrac{I_O\,(t_f\ -\ t)}{C}$ Linearly decaying capacitor voltage with constant current load between (t_f - t). Based on the fundamental relationship that $i_C = C\ dv/dt$.

Note, for practical purposes the the methods presented for calculating v_C all provide reasonable results (within a couple of percent), based on R_LC > 5R_SC.

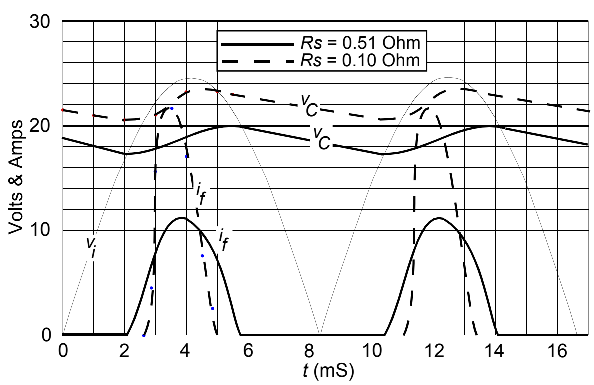
Relationship of Voltages and Currents to R_s, R_L, C.



Reference example in Figure 10.

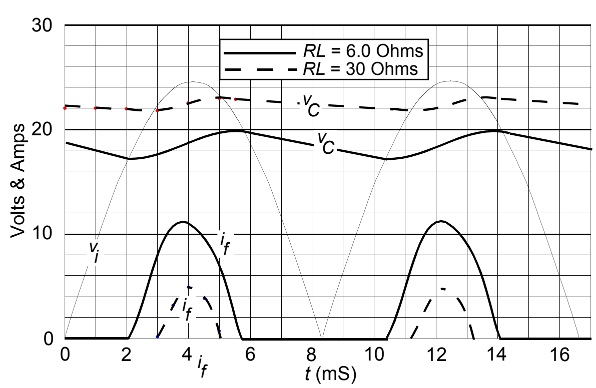
C	1500 μF	6500 μF
v_r	9.25 *Vpp*	2.46 *Vpp*
v_o	17.09 *Vrms*	18.27 *Vrms*
$i_f\,(rms)$	4.28 *A*	4.67 *A*
$i_f\,(pk)$	9.81 *A*	11.16 *A*

Figure B-1. Effect of capacitor value with all other circuit values equal.



Rs	0.1 Ohm	0.51 Ohm
v_r	3.51 *Vpp*	2.46 *Vpp*
v_o	21.37 *Vrms*	18.27 *Vrms*
$i_f\,(rms)$	6.89 *A*	4.67 *A*
$i_f\,(pk)$	21.81 *A*	11.16 *A*

Figure B-2. Effect of series resistance with all other circuit values equal.



R_L	30 Ohm	6 Ohm
v_r	0.82 *Vpp*	2.46 *Vpp*
v_o	21.58 *Vrms*	18.27 *Vrms*
$i_f\,(rms)$	1.53 *A*	4.67 *A*
$i_f\,(pk)$	4.63 *A*	11.16 *A*

Figure B-3. Effect of load resistance with all other circuit values equal.

Appendix B

Relationship of Voltages and Currents to R_s, R_L, C.

Figures B-1 through B-3 expand upon the general statements made in the text regarding the rectifier relationships of voltages and currents to circuit values. These relationships are often misunderstood and often the subject of speculation. With the equations presented in the text, you can now explore these relationships to your own satisfaction. A few more observations that I will offer follow.

As Figure B-1 shows, the ripple voltage is primarily effected by the value of C and R_L. Comparing the 1500 μF and 6500 μF values, the ripple voltage changes by a factor of 3.76, but $i_f\,(pk)$ only changes by a factor of 1.14 and the output voltage only by 1.07. So clearly the capacitor (for a given load resistance) primarily effects the ripple voltage.

In Figure B-2 the effect of series resistance is seen. The regulation is improved from 34% to 14.5% with 0.1 Ohm vs 0.51 Ohm, but $i_f\,(pk)$ increases by a factor of 1.95 and the ripple increases by 1.42. So, as stated in the text, improved regulation is obtained at the expense of higher peak currents.

Finally, Figure B-3 confirms ripple voltage is primarily controlled by $\omega R_L C$ as once again it is seen that an output R_L of 30 Ohms vs 6 Ohms improves the ripple voltage by a factor of 3. Also note the voltage regulation improves to 13.4%, confirming that it is related primarily to the ratio of R_L/R_S.

Appendix C

Fourier Series for Full-wave Rectifier

A full-wave rectified sine wave can be mathematically represented as a Fourier series. [1] While the study of Fourier series analysis is beyond the scope of this document, it is instructive to consider Figure C-1, where the coordinate axes are chosen as shown. From this figure and Equations (C-1) and (C-2) it is apparent all sine terms are zero and the function of $f(\omega t)$ is even. Furthermore, Figure C-1 has a DC component consisting of the first term of Equation (C-2) and is assigned a relative value of 1. The second term, with a magnitude of 2/3, is the second harmonic, 2ω. The succeeding terms of this infinite series are all even harmonics and each become smaller, soon to the point of insignificance in practical circuits.

Equation (C-1).

$$f(\omega t) = \frac{2V_m}{\pi} - \frac{4E_m}{\pi}\sum_{n\ \text{even}} \frac{\cos n\omega t}{(n+1)(n-1)}$$

Equation (C-2)[2].

$$v = \frac{2}{\pi}\ V_m\left(1 - \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t - \frac{2}{35}\cos 6\omega t - \cdots\right)$$

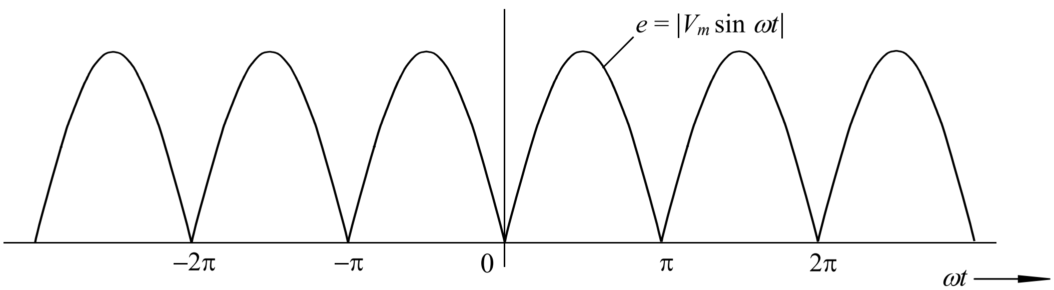


Figure C-1. A full-wave rectified sine wave.

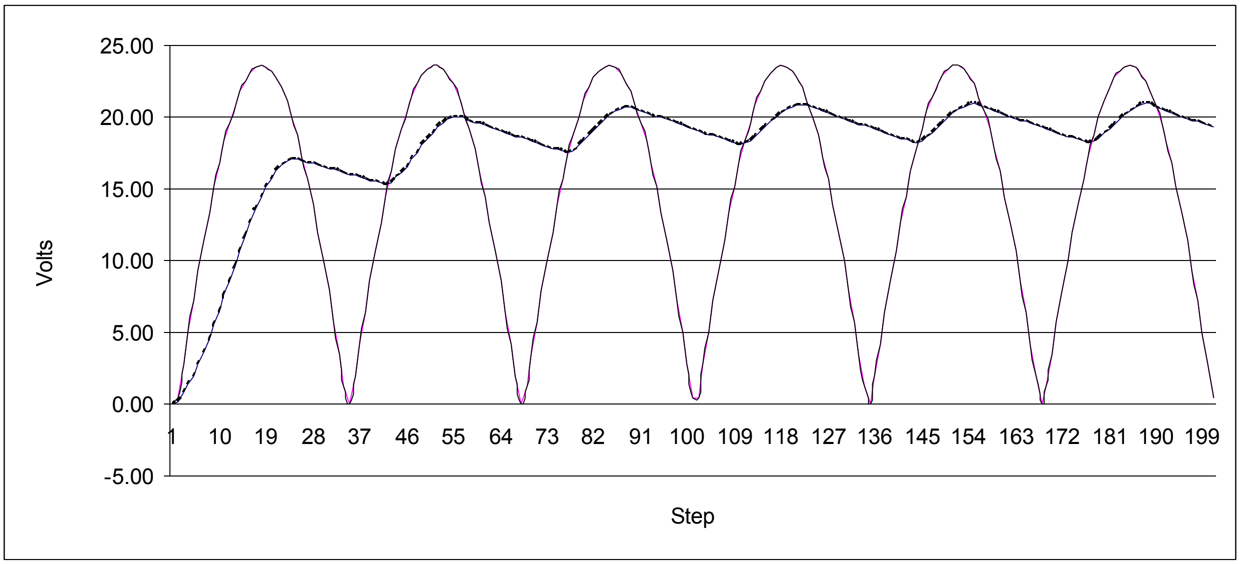
1 *Basic Electronics for Engineers and Scientists* by Lueg and Reinhard 1972 International Textbook Company, Appendix D.

2 *Circuits, Devices, and Systems*, 2nd Printing 1968, Ralph J. Smith, John Wiley & Sons, Inc.

Appendix D

Example Excel Spreadsheet for Figure 10.

Input Circuit Values									
Vi	RL	C	f	Rs'	Rd	Rw	Rc	Vf	n
18.56	6.00	6.50E-03	60	0.460	0.020	0.001	0.040	0.75	1
Calculated rms values					Calculated min/max				
If	Id	Ic	Io	Vo	If(pk)	Vr(pp)	Vr(min)	Vr(max)	Step size
4.960	3.507	4.174	3.276	19.58	11.675	2.72	18.20	20.92	2.50E-04
					Last cycle values				
			Initial Vc:	0.00		Vc(start):	19.28	Vc(end):	19.28



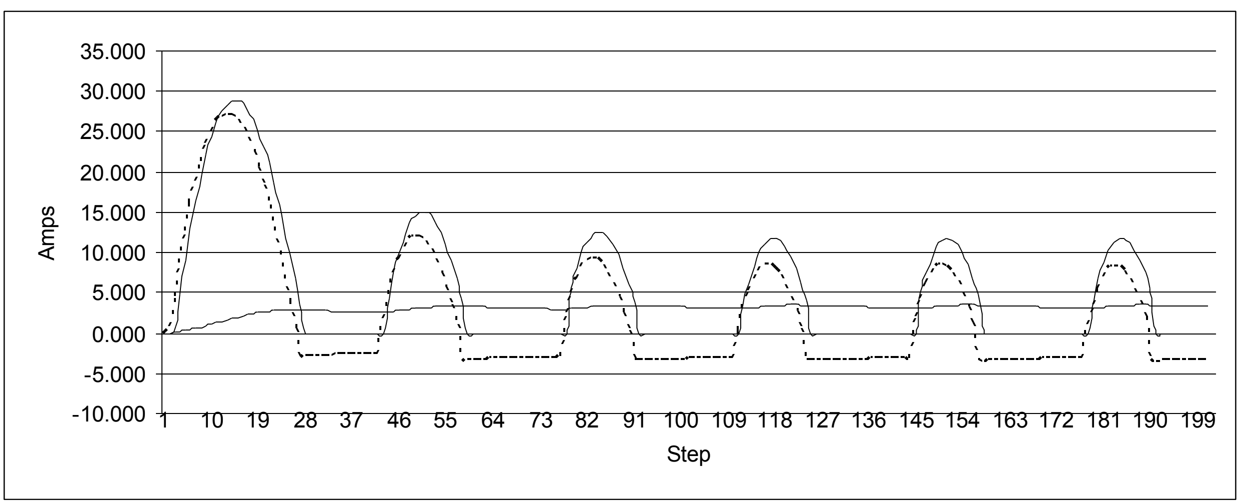


Figure 19. Excel Spreadsheet calculations for example in Figure 10.

Appendix E
Excel Spreadsheet Instructions

Date Run m/d/year	Rectifier Calculator FWCT and FWBR with shunt-capacitor filter	W5BWC Electronics Rev. A 11/5/09

	A	B	C	D	E	F	G	H	I	J
3	Input Circuit Values									
4	*Vi*	*RL*	*C*	*f*	*Rs'*	*Rd*	*Rw*	*Rc*	*Vf*	*n*
5										
6	Calculated *rms* values					Calculated min/max				
7	*If*	*Id*	*Ic*	*Io*	*Vo*	*If(pk)*	*Vr(p-p)*	*Vr(min)*	*Vr(max)*	Step size
8										
9							Last cycle values			
10				Initial *Vc*:			*Vc*(start):		*Vc*(end):	

Plot area for input and output voltages.

Plot area for input, capacitor and output currents.

Cell	Enter	Units
A5	*Vi*	Volts *rms*
B5	*RL*	Ohms
C5	*C*	Farads
D5	*f*	Hz
E5	*Rs'*	Ohms
F5	*Rd*	Ohms
G5	*Rw*	Ohms
H5	*Rc*	Ohms
I5	*Vf*	Volts
J5	*n*	1 = CT, 2= BR
E10	*Vc*	Initial voltage

Table E1. Enter the circuit values under investigation into the above described cells of the Excel Spreadsheet. The results will automatically display in "Calculated" cells and plot in the two plot areas.

Value	Cell	'=	Results
If	A8	'=	Sheet 2!AT208
Id	B8	'=	IF(J5=1,A8/SQRT(2),A8
Ic	C8	'=	Sheet2!AV208
Io	D8	'=	Sheet2!AU208
Vo	E8	'=	Sheet2!AW208
If(pk)	F8	'=	Sheet2!AM207
Vr(p-p)	G8	'=	Sheet2!AK212
Vr(min)	H8	'=	Sheet2!AK210
Vr(max)	I8	'=	Sheet2!AK207
Step size	J8	'=	Sheet2!L5
Last cycle start voltage	H10	'=	Sheet2!AK172
Last cycle end voltage	J10	'=	Sheet2!AK205

Table E2. Map of "Calculated" values.

Appendix E

Excel Spreadsheet Instructions

Table E1. lists the input values required for analysis. These parameters are explained in the text, but further explanation is offered for cell E10, initial voltage. The first run should be made with a value of 0 Volts. This will show the peak input current when the shunt filter capacitor is completely discharged. Note: this is NOT the worst case inrush current - see the text for further explanation.

Compare the "Last cycle" start and end Voltages, cells H10 and J10. If they are within a percent or so of one another the system reached steady state in the first six half-cycles. If not, input Vo into the Initial Volts, cell E10, for the second run and repeat the process until steady state is reached.

Steady state conditions must be established to calculate realistic *rms* values.

Appendix E
Excel Spreadsheet Instructions

Spreadsheet cells and formulas - Sheet 2

Value	Cell	‘=	Results	Notes
Vi	A5	‘=	Sheet 1!A5, A6=A5.....An=An-1	
RL	B5	‘=	Sheet 1!B5, B6=B5.....Bn=Bn-1	
C	C5	‘=	Sheet 1!C5, C6=C5.....Cn=Cn-1	
f	D5	‘=	Sheet 1!D5, D6=D5.....Dn=Dn-1	
Rs’	E5	‘=	Sheet 1!E5, E6=E5.....En=En-1	
Rd	F5	‘=	Sheet 1!F5, F6=F5.....Fn=Fn-1	
Rw	G5	‘=	Sheet 1!G5, G6=G5.....Gn=Gn-1	
Rc	H5	‘=	Sheet 1!H5, H6=H5.....Hn=Hn-1	
Vf	I5	‘=	Sheet 1!I5, I6=I5.....In=In-1	
n	J5	‘=	Sheet 1!J5, J6=J5.....Jn=Jn-1	
ω	K5	‘=	2*PI( )*D5	
ts	L5	‘=	6/(400*D5)	
Re	M5	‘=	(E5+J5*F5+G5)*B5/(E5+J5*F5+G5+B5)	
t	N5	‘=	0, N6=N5+L6	
ve	O5	‘=	0, O6=(SQRT(2)*A6*ABS(SIN(K6*(N6-L6/2)))-J6*I6*B6/(E6+J6*F6+G6+B6)	
ve>0	P5	‘=	0, P6=IF(O6<0,0,O6)	
ici	Q5	‘=	0, Q6=(P6-AJ5)/(M6+H6)	θ_{CI} to θ_{CO}
ic on	R5	‘=	0,R6=IF(P6>AJ5,Q6,0)	θ_{CI} to θ_{CO}
vci	S5	‘=	0,S6=AJ5+R6*L6/C6	θ_{CI} to θ_{CO}
vc on	T5	‘=	0,T6=IF(P6>AJ5,S6,0)	θ_{CI} to θ_{CO}
voi	U5	‘=	0, U6=R6*H6+T6	θ_{CI} to θ_{CO}
vo on	V5	‘=	0, V6=IF(P6>AJ5,U6,0)	θ_{CI} to θ_{CO}
ioi	W5	‘=	0,W6=V6/B6	θ_{CI} to θ_{CO}
io on	X5	‘=	0, X6=IF(P6>AJ5,W6,0)	θ_{CI} to θ_{CO}
ifi	Y5	‘=	0, Y6=R6+X6	θ_{CI} to θ_{CO}
if on	Z5	‘=	0, Z6=IF(P6>AJ5,Y6,0)	θ_{CI} to θ_{CO}

Spreadsheet cells and formulas - Sheet 2 (continued)

Value	Cell	‘=	Results	Notes
icii	AA5	‘=	0, AA6=AJ5/(H6+B6)	θ_{CO} to θ_{CI}
icoo	AB5	‘=	0, AB6=IF(P6>AJ5),0,AA5	θ_{CO} to θ_{CI}
ve	AC5	‘=	P5	θ_{CO} to θ_{CI}
t	AD5	‘=	0, AD6=N6	θ_{CO} to θ_{CI}
vooi	AE5	‘=	0, AE6=AJ5*B6/(B6+H6)	θ_{CO} to θ_{CI}
vooo	AF5	‘=	0, AF6=IF(P6>AJ5,0,AE6)	θ_{CO} to θ_{CI}
vcoi	AG5	‘=	0, AG6=AJ5-AB6*L6/C6	θ_{CO} to θ_{CI}
vco	AH5	‘=	0, AH6=IF(P6>AJ5,0,AG6)	θ_{CO} to θ_{CI}
ic	AI5	‘=	0, AI6=R6+AB6	total
vc	AJ5	‘=	0, AJ6=T6+AH6	total
vo	AK5	‘=	0, AK6=V6+AF6	total
vo	AK207	‘=	MAX(AK173:AK205)	vo (max)
vo	AK210	‘=	MIN(AK173:AK205)	vo (min)
vr	AK212	‘=	AK207-AK210	vr (p-p)
io	AL5	‘=	0, AL6=X6+AB6	total
if	AM5	‘=	0, AM6=Z6	total
if	AM207	‘=	MAX(AM173:AM205)	if (pk)
ic	AN5	‘=	0, AN6=IF(P6>AJ5,AI6,-AI6)	ic corrected for polarity

Appendix E
Excel Spreadsheet Instructions

Spreadsheet cells and formulas - Sheet 2 (continued)

Value	Cell	‘=	Results	Notes
if	AO173	‘=	AM173 to AM205	Selects last 32 steps
n (rms calc)	AP173	‘=	IF(AM173>0,1,0)	Counts # of steps for io > 0
n	AP206	‘=	SUM(AP173:AP205)	Total n with io > 0
2if	AQ173	‘=	IF(AP173>0,2*AM173,0)	Accumulates 2if
	AR173	‘=	IF(AP172=0,-AO173,0)	Subtracts 1st value from 2x1st value
	AS173	‘=	IF(AP173=0,-AQ172/2,0)	Subtracts ½ last entry
if (Amp-sec)	AT173	‘=	AQ173+AR173+AS173	Totals area
	AT206	‘=	SUM(AT173:AT205)	
	AT207	‘=	AT206/(2*AP206)	Area/2x(number of steps)
If (rms)	AT208	‘=	AT207*(SQRT(AP206*L207*2*D207))	
	AU173	‘=	AL173	
from	AU174	‘=	AL174*2	
..........to	AU204	‘=	AL204*2	
	AU205	‘=	AL205	
	AU206	‘=	SUM(AU173:AU205)	
Io (rms)	AU208	‘=	AU206/64	Area/2x(number of steps)
	AV173	‘=	AI173	
from	AV174	‘=	2*AI174	
..........to	AV204	‘=	2*AI204	
	AV205	‘=	AI205	
	AV206	‘=	SUM(AV173:AV205)	
Ic (rms)	AV208	‘=	AV206/66	
	AW173	‘=	AK173	
from	AW174	‘=	2*AK174	
.........to	AW204	‘=	2*AK204	
	AW205	‘=	AK205	
	AW206	‘=	SUM(AW173:AW205)	
Vo (rms)	AW208	‘=	AW206/64	

Appendix F
Components

Transformer primer

While comprehensive transformer design is beyond the scope of this document, the following will provide a starting point for transformer selection or design. As mentioned in the text, rectifier transformers are stressed by the total VA into the primary winding and operate under rather poor *pf* (power factor) conditions, especially with rectifiers feeding into shunt-capacitor filters.

Without delving into a complex analysis of the primary current a reasonable estimate of power factor can be determined by

$$pf = P/VA$$

where the *pf* (expressed as a decimal fraction less than 1.0), $P = (V_O)(I_O)$ - that is the rms output voltage and current of the rectifier, and $VA = (V_i)(I_f)$ - that is the rms input voltage and **total** *rms* secondary current. [1]

For example, a rectifier with an output of 19.5 V*rms* at 3.3 A*rms* using a transformer secondary of 18.6 V*rms* at a total secondary current of 4.6 Ar*ms*, will have a *pf*,

$$pf = (19.50)(3.30)/(18.60)(4.60) = 64.30/85.60 = 0.75.$$

This poor *pf* contributes to transformer heating beyond that encountered with resistive loads and makes the ability to calculate the above voltages and currents important.

While this characteristic has been scrutinized by listing and regulatory agencies, it is not unique to linear power supplies. SMPS that have become mainstream, operate by first rectifying the incoming AC power line into a high DC voltage to operate the switching transformer that provides isolation and voltage scaling. SMPS would have equally poor power factor if not for active *pf* correction circuitry normally included on the front end.

Selection of catalog parts should begin by using the rms values calculated with the Excel spreadsheet to locate parts with target voltage and current ratings, this includes *VA* rating. Next the transformer's total R_S is needed, but not normally specified. Options here are to contact the manufacturer and request the typical values or measure sample parts yourself. Renco Electronics graciously supplied representative data for select parts from their RL-2260 series transformers, see table in Figure F-1. [2]

The leakage inductance is normally insignificant to the results of the analysis presented in this document based on modern transformer designs. For example the RL-2260 parts are constructed of grain-oriented silicon steel, M6 14 mil laminations with 0.66W/lb. core loss at 15 KG. Even with split bobbins, this

1 *Circuits, Devices, and Systems*, 2nd Printing 1968, Ralph J. Smith, John Wiley & Sons, Inc.

2 Transformer information for RL-2260 series curtsey of Renco Electronics, see rencousa.com

Appendix F
Components

Renco RL-2260 Data					
VA and Sec. Voltage	Renco P/N	Individual winding resistance (Ω)			
		Sec.1	Sec. 2	Pri. 1	Pri. 2
43 VA 16 V	RL-2260-43-16	0.156	0.186	26.8	27.11
43 VA 24V	RL-2260-43-24	0.336	0.414	26.77	26.7
43 VA 36V	RL-2260-43-36	0.765	0.907	26.62	26.8
43 VA 230V	RL-2260-43-230	33.29	39.36	26.65	26.8
80VA 16V	RL-2260-80-16	0.065	0.075	10.31	10.38
80VA 24V	RL-2260-80-24	0.153	0.179	10.37	10.47
80VA 36V	RL-2260-80-36	0.353	0.409	10.35	10.39
80VA 230V	RL-2260-80-230	12.13	14.21	10.42	10.4
175VA 16V	RL-2260-175-16	0.0234	0.026	3.24	3.27
175VA 24V	RL-2260-175-24	0.0491	0.057	3.18	3.19
175VA 36V	RL-2260-175-36	0.105	0.124	3.208	3.19
175VA 230V	RL-2260-175-230	3.68	4.38	3.25	3.23

Figure F-1. Transformer winding resistance data courtesy Renco Electronics.

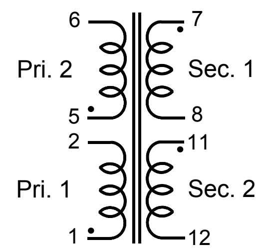


Figure F-2. Renco RL-2260.
Each primary winding rated at 115 VAC, 50/60 Hz.

core minimizes leakage inductance and Renco states it is not a measured nor specified parameter for 60 Hz transformers.

I have measured similar (though layer wound) transformers and found leakage inductance of 0.11 mH, secondary inductance of 28.5 mH and primary inductance of 1.14 H. The 0.11 mH leakage inductance is indeed insignificant for this analysis. With other factors being equivalent, a bobbin wound transformer will have 25 to 50% more leakage than a layer wound, but even this is still an insignificant factor.

The other troublesome transformer design requirement is maximum operating temperature. The manufacturers advertise their product meets UL, CSA, VDE, IEC or other listing agency specifications that include temperature limits for the insulating materials used. They fit their products into neat classes such as IEC/EN 61558-1-2 class A, E, B, F or H for incrementally increasing maximum temperature rise capabilities. But, actually determining the rise in a specific design given specified ambient conditions is rather complex.

When I designed switch mode telcom power supplies for Rockwell-Collins, I had a team of Mechanical Engineers that handled the thermal design and even though I was involved I never became a proficient thermal design guy.

I refer to *Transformer and Inductor Design Handbook* by Colonel Wm. T. McLyman, Jet Propulsion Laboratory © by Marcel Dekker, Inc. for guidance. He has some thermal design information of EI core transformers. Regardless, a design needs to ensure the transformer's class rating is not exceeded for the worst case operating conditions.

REVISION HISTORY			
REV	DATE	PAGES	DESCRIPTION
A	4-15-09	31	Correct spelling
B	11-5-09	28	Add initial Voltage to spreadsheet.